오프라인 경찰수에 대한 밀착상한: 경찰이 도둑보다 빠른 경우

(The Tight Upper Bound on the Offline Cop Number: In the Case Where the Cops Are Fast)

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요 약 지정된 경로를 따라 $n \times n$ 격자 그래프를 탐색하는 경찰들이 있다. 도둑은 이들의 경로를 미리 모두 알고 있지만, 경찰은 도둑의 위치에 대해 전혀 알지도 못하고, 경로를 수정할 수도 없다. 그러면 유한한 시간 내에 도둑을 잡기 위해서는 몇 명의 경찰이 필요할까? 이 숫자, 즉 도둑을 잡기위해 필요한 최소의 경찰수를 *오프라인 경찰수* 라고 한다. 오프라인 경찰수에 대해 알려진 자명한 상한은 n인데, 왜냐하면 n명의 경찰이 일렬횡대로 서서 격자그래프를 바닥부터 천장까지 탐색해간다면 도둑은 어떻게 해도 n 시간 내에 잡힐 수밖에 없기 때문이다. Brass et al. [1]은 교번모델 – 매시간 먼저 경찰이 최대 $s(\geq 1)$개의 간선을 이동한 후 도둑이 최대 한 개의 간선을 이동하는 모델 – 에 대해서, 도둑은 언제나 $\lfloor n/(s+1) \rfloor$ 명의 경찰을 피할 수 있으며, $\lceil n/(s+1) \rceil +1$명의 경찰이 어떠한 도둑도 잡을 수 있도록 하는 탐색경로를 제시했다. 즉, 오프라인 경찰수의 하한이 $\lfloor n/(s+1) \rfloor +1$이고 상한이 $\lceil n/(s+1) \rceil +1$임을 증명했다. 또한 $s=1$인 경우, $\lfloor n/2 \rfloor +1$명의 경찰이 도둑을 잡는 탐색경로를 제시함으로써, $s=1$인 교번모델의 오프라인 경찰수가 $\lfloor n/2 \rfloor +1$임을 보였다. 이 논문에서는 나머지 경우, 즉 $s \geq 2$인 교번모델에서, $\lfloor n/(s+1) \rfloor +1$명의 경찰이 어떠한 도둑도 잡을 수 있도록 하는 탐색경로를 제시하여 오프라인 경찰수가 $\lfloor n/(s+1) \rfloor +1$임을 보인다.
키워드: 추적과 침투, 경찰과 도둑, 안전 경로 계획, 그래프 탐색

Abstract Imagine that cops patrol the $n \times n$ grid on fixed routes. The robber has full knowledge of the cops' routes in advance, but the cops know nothing of the robber's position and cannot change their routes. Then how many cops are necessary to catch the robber in finite time? This number, the minimum number of cops needed to catch the robber is called the *offline cop number*. A trivial upper bound is n since n cops standing abreast and sweeping the grid from the bottom to the top will catch any smartest robber within n time in spite of knowing nothing of the robber's position. In an alternating-move-model where at each time the cops first move (at most) $s(\geq 1)$ edges at a time and then the robber moves at most one edge, Brass et al. [1] proved a lower bound of $\lfloor n/(s+1) \rfloor +1$ and an upper bound of $\lceil n/(s+1) \rceil +1$; they showed that a robber can always escape detection of $\lfloor n/(s+1) \rfloor$ cops indefinitely and provided a strategy for $\lceil n/(s+1) \rceil +1$ cops to catch the robber. They closed the gap in the case where $s=1$, by giving a strategy for $\lfloor n/2 \rfloor +1$ cops to catch the robber. In this note, we close the gap for the other case where $s \geq 2$ by presenting a strategy for $\lfloor n/(s+1) \rfloor +1$ cops to catch the robber.
Keywords: pursuit and evasion, cops and robber, safe path planning, graph search

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1. Introduction

Pursuit-evasion problems have been studied over centuries under many names, like cops and robber [2~4], lion and man [5~7]. In each case, there is a robber and one or more cops; both robber and cops move, and the cops aim to catch the robber. The problems differ by many factors: by the domain of the movement, which is discrete on a graph [7~10] or continuous in the entire plane [5], or in some bounded region [2,3,11,12], by relative speed of robber and cops [1,13,14], by the information that the cops know about the position of the robber completely [5,8-10,13,14], approximately[11], or be constrained by visibility [12,15].

Most of the above discrete problems are modeled as *online games* of two players on a graph, where a player first moves the cops and then the other player moves the robber (which we call *alternating-move*). Both players are assumed to have complete information on where their opponents are at each time. A robber is "caught" by a cop if they are at the same vertex at some time, and the main goal is to identify the so-called *cop number*, the minimum number of cops needed to catch the robber; one cop for a tree, $\lfloor (n+1)/2 \rfloor$ cops for the Cartesian product of n trees [4] and three cops for any planar graph [10]. Further details will be introduced later.

In this paper, we study an offline variant of this problem: given an $n \times n$ grid G_n, the cops patrol along fixed routes and the robber has full knowledge of the cops' routes, but the cops know nothing of the robber's position; this is a reasonable model of the real world and was proposed by Dumitrescu et al. [6]. At each time, all the cops move first (at most) one edge of G_n and then the robber moves (at most) one edge of G_n. We say *the robber evades detection* if it can avoid the nodes where the cops are currently occupying or will step into in the next step, for arbitrary long time interval $[0, T]$. Obviously, offline cop number is at most n since if n cops placed on the bottommost row of the grid sweep the grid row by row then any robber cannot escape n cops in spite of knowledge on the cops' paths in advance.

Brass et al. [1] showed that the cop number in this model is $\lfloor n/2 \rfloor + 1$; a robber can forever escape $\lfloor n/2 \rfloor$ cops, and there is a strategy that $\lfloor n/2 \rfloor + 1$ cops can always capture the robber. They also proved that if the cops and robber move at speed $s (\geq 2)$ and one, respectively, then the cop number is at least $\lfloor n/(s+1) \rfloor + 1$ and at most $\lceil n/(s+1) \rceil + 1$; a robber can forever escape $\lfloor n/(s+1) \rfloor$ cops and there is a strategy for $\lceil n/(s+1) \rceil + 1$ cops to catch any robber. Reducing the gap between these bounds is an immediate open problem, which was noted again in [2]. In this paper, we prove the following theorem and close the gap:

Theorem 1. In an alternating-move-model where at each time the cops first move (at most) $s(\geq 2)$ edges at a time in the $n \times n$ grid G_n and then the robber moves at most one edge in G_n, there is a strategy for $\lfloor n/(s+1) \rfloor + 1$ cops to catch the robber. That is, the (offline) cop number is $\lfloor n/(s+1) \rfloor + 1$ in this alternating-move-model.

2. Related Works and Our Contribution

The literature of pursuit/evasion problems is much too broad to be summarized here. Some historical information can be found in Nahin [16], and we introduce here some most relevant works only.

Online pursuit-evasion in a graph. Online games in a graph and the cop number of graphs have been studied extensively last decades. The cop number of a tree and the Cartesian product of n trees are one and $\lfloor (n+1)/2 \rfloor$, respectively [4]. Aigner and Fromme [10] showed that three cops are enough for any planar graph, but Andreae [17] proved a negative result that for every $k \geq 3$ and ℓ there exists a non-planar k-regular graph with cop number at least ℓ. Frankl [18] introduced Meyniel's conjecture, stating that for every connected graph G with n vertices, $O(\sqrt{n})$ cops are enough to win. This is asymptotically tight because Aigner and Fromme [10] showed that the cop number of any graph without 3- or 4-cycles is at least the minimum degree of vertices, and that there exists an n-vertex graph without 3- or 4-cycles with all degrees at least $c\sqrt{n}$. Many researchers have worked on this problem. The best known upper bound so far is $n2^{-(1-o(1))\sqrt{\log_2 n}}$ [19]. Recently, Fomin et al.

[20] proposed a variety of online game, asking for the cop number of n-vertex graph where the speed of the robber and cops are $s \geq 1$ and one, respectively. Alon and Mehrabian [13,14] showed that the cop number can be as large as $\Omega(s/(s+1))$. Interestingly, in online games of this form, the opposite case where the cops are fast is not of concern because one cop is always enough to catch a robber in a connected graph; this is big difference from the offline analogue that we will study in the sequel.

Offline pursuit-evasion in a graph. Apart from the online games, Dumitrescu et al. [6] introduced an offline variant in the simultaneous-move-model; "offline" means that as explained before, the cops' routes are completely known in advance to the robber over any given arbitrary long time interval $[0, T]$, but here the cops and robber move to their neighboring vertices "simultaneously" at each time. They asked what is the maximum number of cops that a robber can evade on either an $n \times n$ discrete grid or on an $n \times n$ (continuous) square region, and proved that the robber can always escape $\Theta(\sqrt{n})$ offline cops on the $n \times n$ grid, thus provided a lower bound of $\Omega(\sqrt{n})$.

The gap between this lower bound and the trivial upper bound n in the alternating-move-model was closed by Brass et al. [1]; using some discrete isometric theorem, the authors proved that a robber can always escape $\lfloor n/2 \rfloor$ cops, and provided a strategy for $\lfloor n/2 \rfloor + 1$ cops to catch the robber. Note that the lower bound of $\lfloor n/2 \rfloor + 1$ in the alternating-move-model improves the lower bound of $\Omega(\sqrt{n})$ in the simultaneous-move-model given in [6], since for the robber escaping the cops in the alternating-move-model is harder than in the simultaneous-move-model. However their strategy of cops does not provide one in the simultaneous-move-model, thus reducing the gap between $\lfloor n/2 \rfloor + 1$ and n in the simultaneous-move-model remains open, as noticed in [1-3].

Our contribution. We study here offline cop number of $n \times n$ grid graph in the alternating-move-model where first the cops move at most $s(\geq 2)$ edges at a time and then the robber moves at most one edge. Brass et al. [1] proved that a robber can forever evade detection of $\lfloor n/(s+1) \rfloor$ cops and there is a strategy for $\lceil n/(s+1) \rceil + 1$ cops to catch any robber; thus the cop number is at least $\lfloor n/(s+1) \rfloor + 1$ and at most $\lceil n/(s+1) \rceil + 1$. In this note we close the gap by presenting a strategy for $\lfloor n/(s+1) \rfloor + 1$ cops to catch the robber. Note that in online game model, if the cops are fast, then one cop is enough to catch the robber in any connected graph, so this case is interesting only in offline problem. We also provide some observations and open problems for the opposite case where a robber can move $s(\geq 2)$ edges at a time while the cops can move one edge (Section 4). Unlike the online counterpart, this is the first study on the offline cop number with the robber being fast.

3. Proof of Theorem 1

We present a strategy for $\lfloor n/(s+1) \rfloor + 1$ offline cops to catch the robber when the cops can move $s(\geq 2)$ edges at a time while a robber can move one edge in the alternating-move-model. Brass et al. [1] showed that a robber can evade detection of $\lfloor n/(s+1) \rfloor$ offline cops, so this is the best possible. Our strategy is basically the same as in Brass et al. [1], i.e., expanding the set of protected vertices one by one and row by row, and besieging the robber to the upper rows, but unlike their strategy for $s \geq 2$ we fully exploit the fact that at each time the total number of vertices swept by $\lfloor n/(s+1) \rfloor + 1$ cops is strictly greater than n, the width of the grid (the number of vertices of one row).

We begin by introducing some definitions and notations. An $n \times n$ grid $G_n = (V, E)$ $(n \geq 2)$ has n^2 vertices with integer coordinate $[1,n] \times [1,n]$. We assume that there are k cops $c_1, \ldots, c_k$ and one robber r, whose initial positions at time 0 are vertices in V and any move either goes to a neighboring vertex or stays at the same vertex. Denote by $c_i(a)$ and $r(a)$ the location of cop c_i and robber r at time a, and by $N_\ell(u)$ the set of all neighboring vertices $(x+i, y+j) \in V$ of $u = (x, y)$ such that i, j are integers satisfying that $|i| + |j| \leq \ell$. Note that $u \in N_\ell(u)$. At time a, first every cop moves from $c_i(a-1)$ to $c_i(a)$ with $c_i(a) \in N_s(c_i(a-1))$ along the prescribed route and then the robber moves from $r(a)$ to a vertex of $N_1(r(a))$. Denote by $\pi_i(a)$ the set of all intermediate vertices of V along the path from $c_i(a-1)$ to $c_i(a)$. For simplicity of notations, let $C(a) := \bigcup_{i=1}^{k} c_i(a)$ and $\Pi(a) := \bigcup_{i=1}^{k} \pi_i(a)$.

Now we are ready to describe the strategy for $\lfloor n/(s+1) \rfloor +1$ cops to catch the robber in the alternating-move-model where first the cops move at most $s(\geq 2)$ edges at a time and then the robber moves at most one edge. Let $X(a)$ be the set of vertices $v \in V$ such that the robber cannot reach without having been detected *by the end of time* a. Then before the robber starts its move of time a, all the vertices of $X(a-1)$, $\Pi(a)$ and $C(a)$ have been *cleared* by the cops, but except those in $C(a)$, they may be contaminated again by the move of the robber. So, we have that $C(a) \subseteq X(a)$, but some vertex of $X(a-1)$ or $\Pi(a)$, unless it is in $C(a)$, might not be contained in $X(a)$. A crucial observation here is that $X(a-1) \cup \Pi(a) \cup C(a)$ is cleared by the cops before the robber starts its move of time a, so any vertex v with $N_1(v) \subseteq X(a-1) \cup \Pi(a) \cup C(a)$ cannot be reached by the robber in its turn of time a, and is contained in $X(a)$. Namely, if $N_1(K) \subseteq X(a-1) \cup \Pi(a) \cup C(a)$, then $K \subseteq X(a)$. So, the cops' strategy at time a is to move so that at least all points of $N_1(X(a-1))$ are either contained in $X(a-1)$ or on the cops' routes through $\Pi(a) \cup C(a)$ (implying that $X(a-1) \subseteq X(a)$) and if possible, to sweep all neighing vertices of one more point for adding it into $X(a)$. In this way, the cops extend $X(a)$ row by row, besieging the robber to the upper rows and finally leave no place for the robber outside $X(a)$.

Refer to Figure 1 for the illustration of the strategy. This figure illustrates the case where $n=14$, $s=4$ since this is the most difficult case for given $s=4$ and three cops; obviously detecting smaller grid with the same number of cops moving at the same speed becomes easier. At time a, solid disks with different colors represent $C(a)$, i.e., cops just moved from the other ends ($C(a-1)$) of the line segments (whose internal vertices are $\Pi(a)$), and the cross-marks represent some vertices in $X(a)$. At time 1, all neighbors of the cross-mark (including itself) is contained in $\Pi(1) \cup C(1)$, so the cross-mark is contained in $X(1)$. At time 2, the upper neighbor and itself of the rightmost cross-mark are contained in $X(1)$ and the left neighbor of the cross-mark is contained in $\Pi(2)$, so the rightmost cross-mark is contained in $X(2)$. The leftmost cross-mark is added into $X(2)$ because the cross-mark itself and the upper and left neighbors are contained in $\Pi(2)$ and the right neighbor is contained in $X(1)$. At time 3, the leftmost cross-mark is contained in $X(3)$ since the cross-mark itself and the upper neighbor are contained in $\Pi(3)$, the left neighbor is in $C(2)$ and thus in $X(2)$, and the right one is in $X(2)$. The other two cross-marks are contained in $X(3)$ because they were in $X(2)$ and all their neighbors are contained in $\Pi(3) \cup C(3)$. At time 4, the four cross-marks are contained in $X(4)$ because all their neighbors including themselves are either contained in $X(3)$ or swept by the cops along $\Pi(4) \cup C(4)$. By repeating the procedure shown in Figure 1 row by row, the cops can sweep over the whole grid without being invaded by the robber.

4. Concluding Remarks

We solved an open problem posed in [1-3], closing the gap between $\lfloor n/(s+1) \rfloor +1$ and $\lceil n/(s+1) \rceil +1$ for the offline cop number of the $n \times n$ grid graph in the alternating-move-model where the cops can move $s(\geq 2)$ edges at a time while a robber can move one edge. We proved that the upper bound is $\lfloor n/(s+1) \rfloor +1$ by presenting a strategy for $\lfloor n/(s+1) \rfloor +1$ cops to catch any robber, thus the offline cop number is $\lfloor n/(s+1) \rfloor +1$ (Theorem 1). Brass et al. [1] showed that when $s=1$ the offline cop number is $\lfloor n/2 \rfloor +1$, so the result here combined with the results in [1] leads to the conclusion that the offline cop number of the $n \times n$ grid graph is $\lfloor n/(s+1) \rfloor +1$ for all $s \geq 1$ in the alternating-move-model where the cops first move at most $s(\geq 1)$ edges at a time and then a robber moves one edge. Note that in the classical online model, if the cops are faster than the robber, then one cop is enough to catch the robber in any connected graph, so this case is interesting only in offline problem.

We close this section with some relevant observations and open problems.

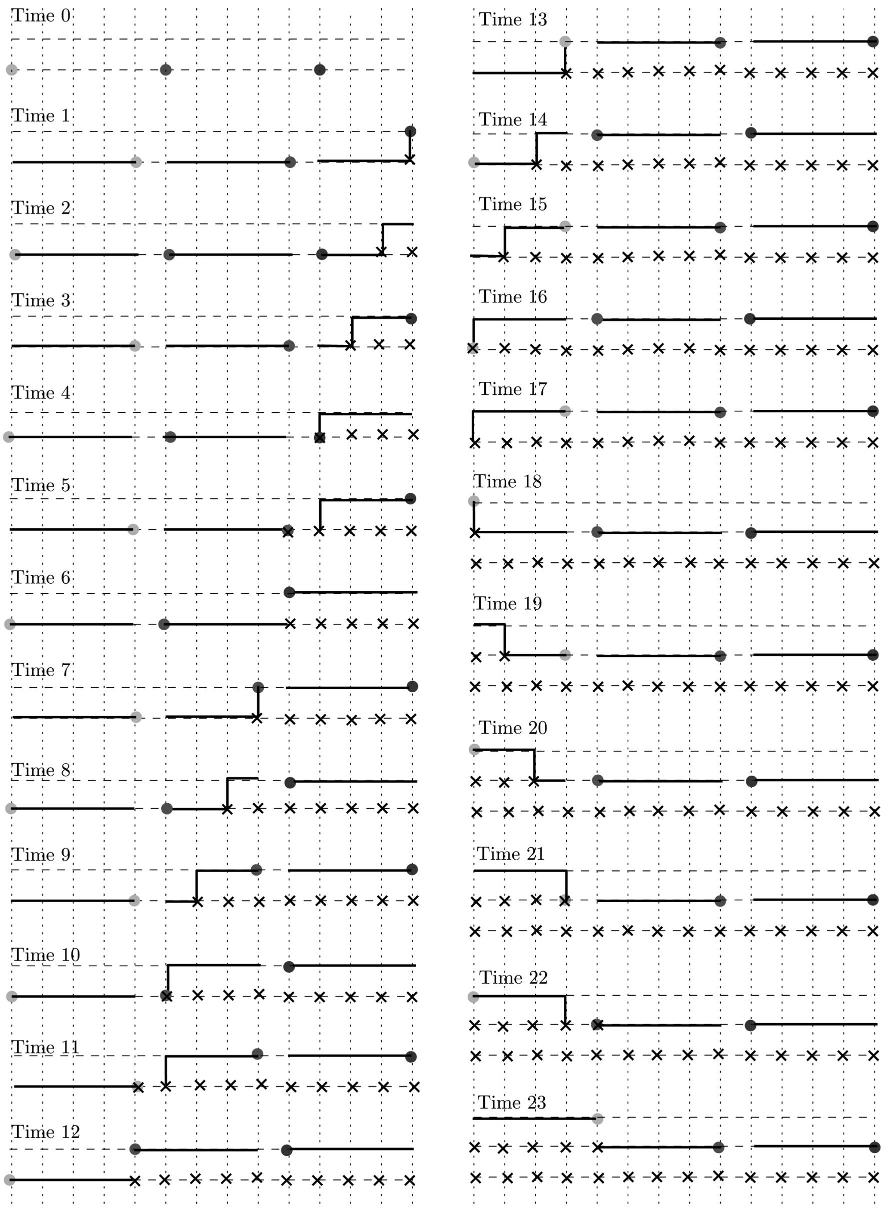

Fig. 1 Illustration of strategy for $\lfloor n/(s+1) \rfloor + 1$ cops to catch a robber when $n=14$, $s=4$. This is the most difficult case for given $s=4$ and three cops since detecting smaller grid with the same number of cops moving at the same speed becomes easier. At time a, solid disks with different colors represent cops just moved from the other ends of the line segments, and the cross-marks represent some vertices in $X(a)$.

Simultaneous-move-model. We noted already in Section 2 that a lower bound in the alternating-move-model provides the same lower bound in the simultaneous-move-model, but the strategy of cops in the alternating-move-model does not necessarily provide one in the simultaneous-move-model. Indeed, the lower bound of $\lfloor n/(s+1) \rfloor + 1$ $(s \geq 1)$ in the alternating-move-model becomes a lower bound for the simultaneous-move-model, but none of the strategies given in Brass et al. [1] or here work in the simultaneous-move-model. So, reducing the gap between $\lfloor n/(s+1) \rfloor +1$ and n is still an interesting open problem. Many researchers conjecture that the offline cop number in the simultaneous-move-model is n [2,3,7].

The case where the robber is fast. It is interesting to study the offline cop numbers in the opposite case where a robber can move $s(\geq 2)$ edges at a time while the cops can move one edge. Unlike the online counterpart, this is the first study on the offline cop number with the robber being fast. The strategy for n cops that gives the trivial upper bound of n in the alternating-move and simultaneous-move models, i.e., standing abreast and sweeping the grid from the bottom to the top, works fine even if the robber is fast. Hence the offline cop number in both models where a robber is fast is at most n. The proof for the lower bound of $\lfloor n/(s+1) \rfloor +1$ $(s \geq 1)$ (given in Brass et al. [1]) works again in both models since if a robber with speed one can escape $\lfloor n/(s+1) \rfloor$ cops then so can a robber with speed $s \geq 2$. Therefore the offline cop number in the alternating- or simultaneous-move-model where a robber is fast is at least $\lfloor n/(s+1) \rfloor +1$. Reducing the gap between $\lfloor n/(s+1) \rfloor +1$ and n is an interesting open problem.

To understand the difference from the case where the speed of cops and robber are both one, refer to Figure 2. It illustrates an example of the cops' move in the alternating-move-model where the cops first move at most one edge and then a robber moves at most two edges. At time a, solid disks with different colors represent $C(a)$, the cross-marks represent some vertices in $X(a)$. All the vertices v with $N_2(v) \subseteq X(a-1) \cup C(a)$ cannot be reached by a robber by the end of time a, so those vertices (indicated by boxes) are contained in $X(a)$. Until time 3, $X(a)$ in this model is the same as that in the model where the speed of cops and robber are both one. But at time 4, after the green cop leaves for vertex $(2,4)$, the vertices $(1,3), (1,4)$ which were contained in $X(3)$ can be occupied by a robber waiting at vertex $(1,5)$, so neither of them are contained in $X(4)$. In the case of both speed one, such vertex as $(1,3)$ would be contained in $X(4)$ and thus indicated by cross-mark.

Back to the open problem, we believe that the offline cop number in both move-models with $s \geq 2$ is n, and present some observations below which might be helpful for the readers to prove it.

Conjecture. In an alternating-move-model where at each time the cops first move (at most) one edge in the $n \times n$ grid G_n and then the robber moves at most $s(\geq 2)$ edges at a time in G_n, a robber can escape $n-1$ cops forever. That is, the (offline) cop number is n in the alternating- or simultaneous-move-model.

Generalizing the commonly used techniques in the proofs of lower bound in [1,3,7], we obtained the following observations in the alternating-move-models the cops first move at most one edge and then a robber moves at most s edges $(s=1,2)$. For any set $A \subseteq G_n$, denote by $I_s(A)$ and $B_s(A)$ the set of vertices $v \in A$ such that $N_s(v) \subseteq A$ and $N_s(v) \not\subseteq A$, respectively, and by A^c the complement of A. Let $\widetilde{X}(a) = X(a-1) \cup C(a)$.

(1) $I_s(\widetilde{X}(a)) \subseteq X(a) \subseteq \widetilde{X}(a)$

(2) $|X(a)| \leq |X(a-1)| + k$, where k is the number of cops.

(3) $X(a) = I_s(\widetilde{X}(a)) \cup C(a) \cup \{v \in B_s(\widetilde{X}(a)) \setminus C(a):$

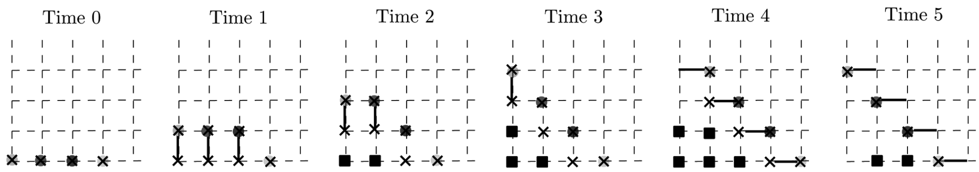

Fig. 2 An example of the cops' move in the alternating-move-model where the cops first move at most one edge and then a robber moves at most two edges (i.e., $s=2$). At time a, solid disks with different colors represent $C(a)$, the cross-marks represent some vertices in $X(a)$, and the boxes all vertices v satisfying that $N_s(v) \subseteq X(a-1) \cup C(a)$ and thus contained in $X(a)$.

$\forall u \in \widetilde{X}(a)^c, \forall \pi(u,v), \pi(u,v) \cap C(a) \neq \varnothing \}$

where $\pi(u,v)$ denotes the shortest path in G_n from u to v.

(4) Let $s=1$. Then the third term in the right-hand side of (3) is an empty set, and we obtain that $|X(a)| \leq |\widetilde{X}(a)| - |B_1(\widetilde{X}(a))| + k \leq |X(a-1)| - |B_1(\widetilde{X}(a))| + 2k$. If we assume that the cardinality of $X(\cdot)$ becomes greater than $n^2/2$ at time a for the first time, then a discrete isometric theorem given in [1,3,7] yields that $|B_1(\widetilde{X}(a))| \geq n$. Thus if $k \leq \lfloor n/2 \rfloor$, we obtain that $|X(a)| \leq |X(a-1)|$, contradicting the assumption and proving the lower bound of $\lfloor n/2 \rfloor + 1$ [1,3,7].

(5) Let $s=2$. Then we obtain that $|X(a)| \leq |X(a-1)| - |B_2(\widetilde{X}(a))| + 2k + |K|$, where K is the third term in the right-hand side of (3). Applying a similar approach, if we assume that the cardinality of $X(a)$ becomes close to $n^2/2$, then a discrete isometric theorem yields that $|B_2(\widetilde{X}(a))| \geq 2(n-1)$. A tedious computation gives that $|K| \leq 2k$. But this approach gives only a lower bound of roughly $n/2$, which was immediately obtained by the result of Brass et al. [1]. as noted above. So, proving Conjecture seems to need other approaches or much better isometric theorem, as was pointed out in Alonso et al. [3].

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